This about Covers It!

Strategies for Finding Area

Many adults, when asked how to find the area of a figure, promptly answer, "Length times width!" They understand area as a formula rather than as a concept—the amount of space covered by the inside boundaries of a two-dimensional figure. The area of a rectangle can be found by using the formula $A = l \cdot w$, but how do you calculate the area of a nonrectangular polygon such as a triangle or an octagon? How about irregular figures that have curved sides? Indeed, the figures found in our everyday world are often irregular. Graphic designers, architects, and engineers are some of the professionals who need to be able to find the area of interesting, irregular figures in their daily work. To do this, they must understand what area is and use creative mathematical ways to arrive at estimates as well as precise answers. So it is very important for teachers to give student mathematicians the opportunity to develop their understanding of area as a measure of covering before teaching them any formula.

Connections to the Standards

With regard to the teaching and learning of measurement, *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics 2000), as set forth in its Measurement Standard, calls for elementary instructional programs that will "enable all students to—

- understand measurable attributes of objects and the units, systems, and processes of measurement; [and]
- apply appropriate techniques, tools, and formulas to determine measurements" (p. 44).

Research suggests that students often struggle with these concepts as they relate to area (Martin and Strutchens 2000). To address these objectives, this investigation asked students to explore ways to estimate the actual area of two irregular figures. Because they had no efficient formulas to use, students discovered and then fine-tuned their own strategies for determining the number of square units within these irregular areas by incorporating methods that made sense to them. They shared their strategies during class discussions, a process that helped them discover and appreciate diverse approaches that included more efficient ways to determine area.

Communication was a central component of this investigation. Students wrote about the strategies they used to find the area of one of the figures. The act of writing about mathematics, as *Principles and Standards for School Mathematics* (NCTM 2000) states in its Communication Standard, "should enable all students to—

Organize and consolidate their mathematical thinking through communication
Communicate their thinking coherently and clearly to peers, teachers, and others
Analyze and evaluate the mathematical thinking and strategies of others
Use the language of mathematics to express mathematical ideas precisely" (p. 194).

The writing also proved beneficial to teachers because it allowed them to formatively assess student understanding, provide individual and differentiated feedback, and adjust and focus future investigations. As a result of working on these

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activities, the students, rather than relying on a formula, deepened their understanding of area as a measure of covering.

This investigation is from the level-3 unit of *What's the Me in Measurement All About?* (Gavin et al. 2006), one of twelve units in the Project M^3: Mentoring Mathematical Minds series created for grades 3–5 students with mathematical talent.

Day 1

Using a fried egg to discover strategies for finding area

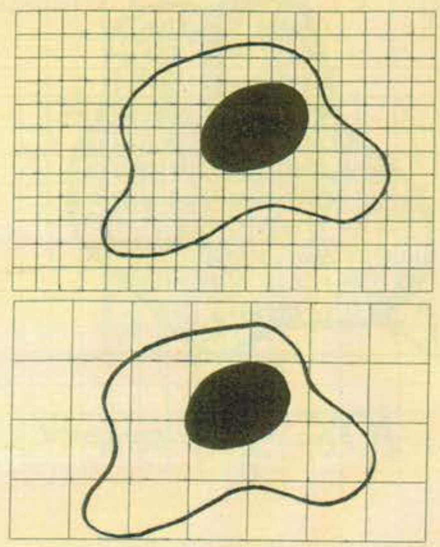
Third-grade students participating in Project M^3: Mentoring Mathematical Minds first discussed the idea that area is the measure of the region inside the boundaries of a two-dimensional or "flat" figure and is typically measured in square units. They considered what a square centimeter looks like (a *square centimeter* is a square that measures 1 centimeter on each side) and then were asked to determine what other units might be used to measure area—square inches, square feet, square meters, square miles, and so forth. This discussion also addressed the notion that the units selected to measure area must reflect the size of the area being measured. For example, square miles would be an appropriate unit for measuring the area of a town, while square inches would be a more appropriate unit for measuring a tabletop.

The teachers explained to the students that they were going to estimate how much area a fried egg covers. After showing students samples of 1-inch, 2-inch, and centimeter grid paper, the students were asked to determine what size grid would give an estimate closer to the actual area. They concluded that the centimeter grid paper would provide the best solution because, by allowing them to break the egg into smaller pieces, it could give a more precise estimate of the pieces near the "edge" of the fried egg that would not completely fill an entire square (see **fig. 1**).

Students then traced the shape of a fried egg on a sheet of centimeter grid paper and estimated, to the nearest square centimeter, the actual area it covered. At the bottom of the grid paper, they recorded the number of square units and shared their strategies with one another. The class discussion revealed that the students attempted this task in many different ways, and this diversity of approach gave teachers the opportunity to informally assess the students' understanding of the strategies for finding the area of irregular shapes.

Figure 1

After superimposing the fried egg shape on different-sized grids, the students concluded that they could provide a more precise estimate of the fried egg area by using centimeter grid paper (a) than by using inch grid paper (b).



a.

b.

Note: Graphics for the figures in this article are not shown actual size.

Figure 2

In determining the area of the fried egg, Katie counted the whole squares but ignored most of the partial squares. (Note the highlighted partial squares.)

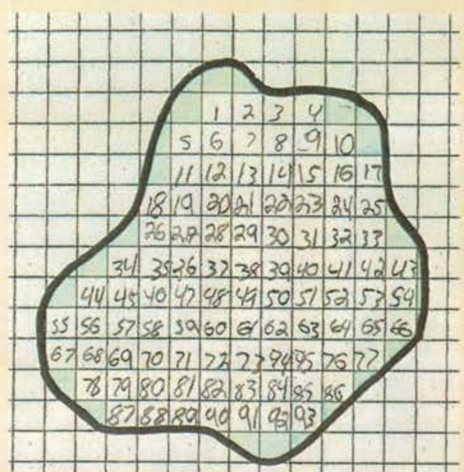


Figure 3

Joe accounted for both whole and partial squares but did not realize that all partial squares were not half squares. (Note the two highlighted partial squares.)

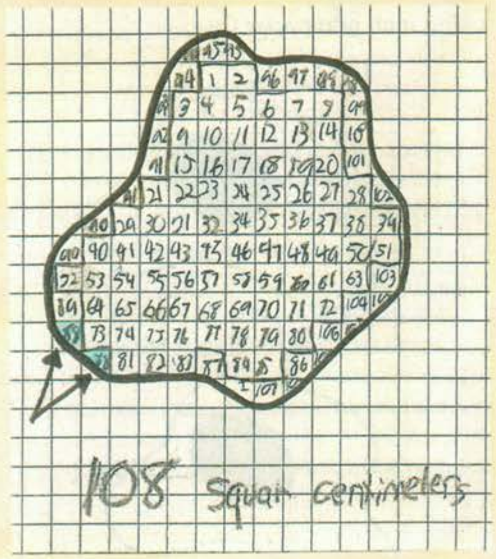


Figure 4

Montana's group used arrays to quickly calculate the area of rectangular shapes within the body tracing.

In order to report the total area, Katie began by counting each square that was wholly covered ("whole squares") but ignored the partially covered squares ("partial squares") within the fried egg shape depicted on the grid paper (see **fig. 2**). She recorded the numbers in the squares as she counted, explaining that this helped her avoid double-counting square centimeters. However, Katie failed to recognize that the partial squares are also an important part of the area within the fried egg shape and must be considered to get a more accurate estimate of the total area. Although her count included the partial squares that covered nearly the whole square, she ignored all other partial squares.

In his calculation of the area within the fried egg shape, Joe did take into account the partial squares. His strategy for counting built on his understanding of fractions. He knew that two halves make a whole, so he looked for two partial squares that could be combined to make a full square centimeter and counted those as one full square (see **fig. 3**). To keep track as he counted, he wrote the same number in both parts to indicate that together they were counted as one square unit.

Although counting individual squares as Katie did was an effective strategy in finding the area of the fried egg, it was not a particularly efficient one. It appeared that many students relied on this strategy because the area covered by the fried egg was relatively small and counting each square was manageable. In addition, most students began to consider what to do with the partial squares. However, some students, like Joe, did not realize that all partial squares were not "half squares." In other words, to calculate the area of irregular shapes, any given number of partial squares—not just two half squares—must be grouped in such a way that they come together like a puzzle to make one full square.

Day 2

Using body tracings to develop more strategies for finding area

Now that the students had a beginning understanding of the strategies for finding the area of irregular shapes, the teachers planned to develop it further by challenging them to apply their strategies to calculate the area of a much larger figure. Having a larger area to account for would establish a need for more efficient strategies for counting whole squares, and using larger square units (i.e., square inches rather than square centimeters) would make it easier for students to visualize the partial squares, encouraging them to use more precisely the strategy of piecing them together to make one whole square.

Groups of three to five students worked together to determine the area of the tracing of a group member who had volunteered to be the model. (It is easier to calculate the area of the body tracing when students, while being traced, spread apart their fingers, place their arms away from their sides, and let their feet turn outward.) Before beginning the activity, the teachers held a brief class discussion to remind students about the strategies they had used to determine the area of the fried egg.

As the students worked, the teachers noticed that groups were using different strategies to determine the area. First, they sectioned the body tracings differently. Some groups measured individual body parts (such as the head, arms, trunk, and legs) and then added the areas of these individual parts together. This was encouraging because it demonstrated that the students understood area as a measure of covering; they obviously felt comfortable cutting the figures apart and then putting them back together. Others used the body's vertical line of symmetry to count the area of half the body and then double that number. (Some students reasoned that this estimate was probably not the most accurate because of their faulty tracing!) Second, the students continued to use a variety of strategies to account for the whole and partial squares. During the ensuing class discussion, the teachers planned to call on groups who had used more efficient and precise strategies.

Montana shared how the students in her group had used their understanding of multiplication to efficiently count some of the square inches (see **fig. 4**). They drew rectangular arrays, and then, rather than individually counting the whole squares inside the arrays, they counted the number of rows and columns and multiplied them to get the total number of square inches. They then added the individual squares and partial squares outside the arrays to arrive at the total area for the body tracing. Montana, in fact, was one of just a few students who had used an array to determine the area of the fried egg, and she noted that the arrays were even more helpful when working with the larger figure.

Navia's group also used arrays to calculate the area but in a different way. Instead of drawing several arrays inside the figure, they drew one large one encasing different body parts, such as the hand (see **fig. 5**). They then counted the number of rows and columns in the array and multiplied to find its area. Finally, to determine the area of the hand, they subtracted the number of full and partial squares outside the hand within the array.

Figure 5

Navia's group used an array outside the hand and then, to determine the area of the hand, subtracted the area outside the tracing.

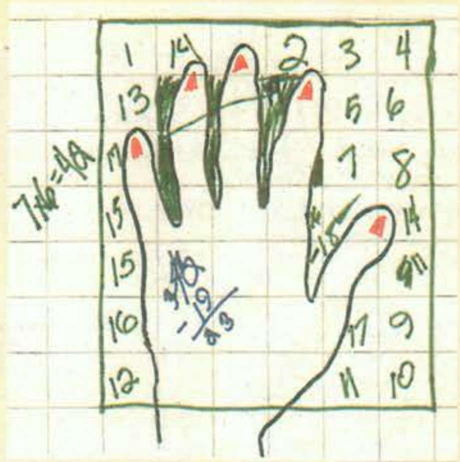
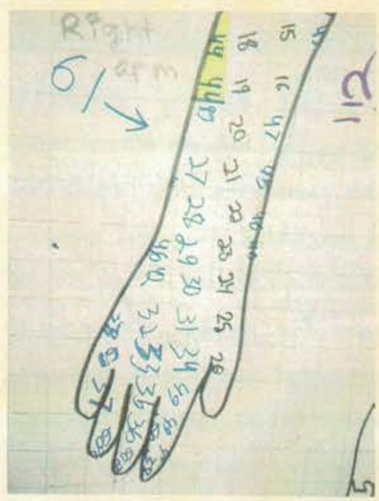


Figure 6

Skyler's group combined partial squares to make one whole square.



The students also more clearly understood how to account for partial squares of areas of different sizes. Skyler explained to the class how his group had taken partial squares from all around the body tracing (e.g., two from the hand and one from the

Figure 7

Miguel's explanation in his journal of the strategies he used to measure the square area of the body tracing (note the highlighted numbers at the bottom of the page)

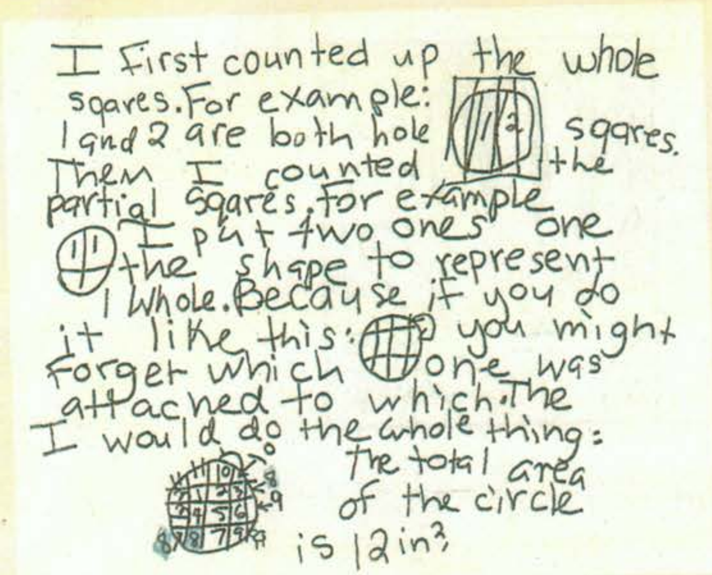

I First counted up the whole
sqares. For example:
1 and 2 are both hole sqares.
Them I counted the
partial sqares, for example
I put two ones one
the shape to represent
1 whole. Because if you do
it like this: you might
forget which one was
attached to which. The
I would do the whole thing:
The total area
of the circle
is 12 in²

Figure 8

Kara's explanation in her journal of the strategies she used to measure the square area of the body tracing

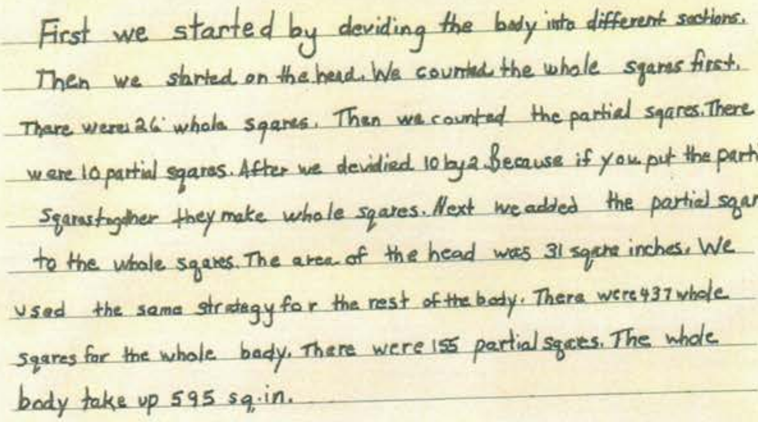

First we started by deviding the body into different sections.
Then we started on the head. We counted the whole sqares first.
There were 26 whole sqares. Then we counted the partial sqares. There
were 10 partial sqares. After we devidied 10 by 2 Because if you put the partial
sqarestogther they make whole sqares. Next we added the partial sqares
to the whole sqares. The area of the head was 31 sqare inches. We
used the same strategy for the rest of the body. There were 437 whole
sqares for the whole body. There were 155 partial sqres. The whole
body take up 595 sq. in.

Figure 9

Teacher's depiction of irregular shapes covering partial squares of the area grid

foot) to make approximately one whole square inch (see **fig. 6**). To keep track of the partial squares, his group adopted Joe's method for the fried egg activity and wrote the same number in each area that went together to make one square inch. Instead of just pairing half squares together, Skyler's group put together partial squares that, for example, had covered approximately 1/4 and 3/4 of the square to make one full square.

Student Mathematicians Write On

It appeared that the students as a class understood and were able to apply different strategies for determining the area of an irregular figure. Now it was time to determine what conclusions individual students had reached. They were asked to explain in writing the strategies they had used to measure the area of their body tracing. Given the class discussions, the teachers anticipated that the students would write about the following strategies for whole squares:

- counting individual whole squares
- using arrays within the figure and then adding other whole and partial squares
- using an array outside the figure and then subtracting other whole and partial squares
- piecing together two half squares to make a whole square
- putting together any number of partial squares to make one whole square

Through notes on each student's written response, the teachers addressed that particular student's understanding of strategies for determining the area and further encouraged his or her use of the strategies. They identified what each student had done to help others be aware of what they understood and also provided guidance to expand on their understanding. Following are a few examples of this feedback:

Miguel included an example of a figure—he drew a circle—to describe his strategies for both the whole and partial squares (see **fig. 7**). However, his grid contained only four whole square units, so it was not necessary for him to use an array. His teacher responded:

> You seem to really understand that partial squares can make whole squares if you make sure to cover every part of the whole square. You

are also making use of a great method to keep track of what partial squares you have already used! For example, I see how an 8 was written on three partial squares. Check to make sure that together they make a whole square. You might want to cut them out and piece them back together to check. I also wonder what strategy you would use to count the whole squares if you had a larger figure. Is there another way that you could count the squares that would be quicker than counting them one by one?

Kara appeared to understand that determining the area of the entire figure involves adding the areas from the whole and partial squares. However, it seemed that she did not have a complete understanding of how to determine the area of the partial squares and assumed that combining any two would result in a whole square (see **fig. 8**). She wrote, "There were 10 partial [squares]. After we [divided] 10 by 2. Because if you put the partial [squares] together they make whole [squares]." Her teacher replied:

> I agree with you—you need to add the area of the partial squares to the area of the whole squares to get the area of the entire figure. I noticed that you put two partial squares together to make one whole square. Does this always work? I wonder how you would figure out the area of shaded parts of the partial squares that look like the ones [in **fig. 9**]?

Miguel and Kara, along with their classmates, continued their written communication with their teachers about their understanding of area even after the lesson had ended. After the teachers had responded to individual students' written thoughts, the students replied to their teachers' queries. This gave students the opportunity to solidify their understanding.

Closing Thoughts

These samples of student work demonstrate how important it is to provide students with ample opportunities to investigate irregular shapes and develop strategies to determine their areas. This investigation allows students to develop a comprehensive understanding of area, including knowing that they can break up an irregular figure into whole and partial square units, use arrays to efficiently calculate rectangular areas, and put together partial squares to make whole squares. As a result of understanding the concept of area as a measure of covering, the students, when asked how to find the area of a figure, now respond, "It depends on the shape of the figure."

References

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Martin, W. Gary, and Marilyn E. Strutchens. "Geometry and Measurement." In *Results from the Seventh Mathematics Assessment of the National Assessment of Educational Progress*, edited by Edward A. Silver and Patricia Ann Kenney, pp. 193–234. Reston, VA: National Council of Teachers of Mathematics, 2000.

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