Journal of Advanced Academics
jaa Volume 18 ❖ Number 4 ❖ Summer 2007 ❖ pp. 566–585

Project M^3:

Mentoring Mathematical Minds—A Research-Based Curriculum for Talented Elementary Students

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More than 25 years ago, the National Council of Teachers of Mathematics ([NCTM], 1980) in their *Agenda for Action* stated, "The student most neglected, in terms of realizing full potential, is the gifted student of mathematics. Outstanding mathematical ability is a precious societal resource, sorely needed to maintain leadership in a technological world" (p. 18). Unfortunately, the results of the Trends in International Mathematics and Science Study ([TIMSS], 2000, 2004) showed that U.S. students continue to fall far below their international peers on the mathematics assessment. In fact, the gap increased from 4th to 12th grade, by which time only two countries had students performing significantly lower than the United States (TIMSS, 2000). The most talented students in the United States also compared unfavorably with their peers. While 40% of eighth-grade students in Singapore and 38% of eighth graders in Taiwan scored

summary

To date, there has been very little research-based mathematics curriculum for talented elementary students. Yet the gifted education and mathematics literature suggest support for curriculum that is both enriched and accelerated with a focus on developing conceptual understanding and mathematical thinking. Project M^3: Mentoring Mathematical Minds is a 5-year Javits research grant project designed to create curriculum units with these essential elements for talented elementary students. These units combine exemplary teaching practices of gifted education with the content and process standards promoted by the National Council of Teachers of Mathematics. The content at each level is at least one to two grade levels above the regular curriculum and includes number and operations, algebra, geometry and measurement, and data analysis and probability. The focus of the pedagogy encourages students to act as practicing professionals by emphasizing verbal and written communication. Research was conducted on the implementation of 12 units in 11 different schools, 9 in Connecticut and 2 in Kentucky. The sample consisted of approximately 200 mathematically talented students entering third grade, most of whom remained in the project through fifth grade. Students in this study demonstrated a significant increase in understanding across all mathematical concepts in each unit from pre- to posttesting. Thus, Project M^3 materials may help fill a curriculum void by providing appropriate accelerated and enriched units to meet the needs of mathematically talented elementary students.

Gavin, M. K., Casa, T. M., Adelson, J. L., Carroll, S. R., Sheffield, L. J., & Spinelli, A. M. (2007). Project M^3: Mentoring Mathematical Minds—A research-based curriculum for talented elementary students. *Journal of Advanced Academics, 18,* 566–585.

at the most advanced level on the 2003 TIMSS mathematics assessment, only 7% of U.S. eight graders scored at this level (TIMSS, 2004). Clearly, U.S. students, including the top ones, are not measuring up internationally.

On the national level, results from the National Assessment of Educational Progress (NAEP) indicated that although student performance increased in mathematics, a large percentage of students still were not performing at an acceptable level (Perie, Grigg, & Dion, 2005). In fact, 70% of U.S. eighth-grade students cannot solve a word problem involving more than one operation. Moreover, there was a frightening shortage of students performing at the highest level. Only 5% of fourth-grade students and 6% of eighth-grade students performed at the "advanced" level. It is at this level that eighth-grade students are expected to use abstract thinking, which is a cornerstone of high-level mathematics. Whether we look at international or national measures, the U.S. system clearly is failing. How can we change this situation to help talented math students, especially those of diverse backgrounds, learn more mathematics and achieve at higher levels?

Curriculum for Mathematically Talented Students

One of the first steps in addressing the needs of these students is to provide effective, high-level curriculum. However, to date there is a paucity of research-based mathematics curriculum for mathematically talented elementary students. Nevertheless, the mathematics and gifted education literature suggests that there may be support for curriculum that focuses on both mathematical content and processes, combines acceleration and enrichment practices, addresses the range and diversity of students' mathematical talents through differentiation, and encourages students to process mathematics in ways similar to those of practicing professionals.

Addressing Mathematical Content and Processes

In the latest reform movement, the National Council of Teachers of Mathematics (2000) has not only outlined *what* students should learn (i.e., the number, algebra, geometry, measurement, and data analysis and probability content standards) but also *how* they should learn mathematical content. The process standards encourage students to problem solve, communicate, reason, make connections, and use different representations as they engage with mathematics. Some elementary mathematics curricula based on the NCTM (2000) standards, including *Math Trailblazers*, *Everyday Mathematics*, and *Investigations in Number, Data, and Space*, have students employ these mathematical processes as they study these content areas. In addition, these curricula are concept-based and focus on significant mathematical ideas. Research on the implementation of these curricula indicates that students using these curricula do as well as other students on traditional measures of mathematics achievement, even on measures of computational skill. Furthermore, on formal and informal assessments of conceptual understanding and ability to solve problems, students using the reform-based curricula generally do better than other students (Carroll & Isaacs, 2003; Carter et al., 2003; Mokros, 2003; Putnam, 2003). Thus, research has shown that curriculum developed using the NCTM (2000) content and process standards is effective. However, these curricula were designed for the general student population and not specifically for talented students.

As with all students, the curriculum used with mathematically talented students should be based on the NCTM (2000) content and process standards, but they also should "explore topics in more depth, draw more generalizations, and create new problems and solutions related to each topic" (Sheffield, 1994, p. 21). In addition, the focus of curriculum for students with mathematical talent should be problem solving (NCTM, 1980, 2000; Sheffield, 1994; Wheatley, 1983). Problem solving is interrelated with the other mathematical processes, which include communication, connections, reasoning, and representations.

A Combination of Acceleration and Enrichment

Research studies on the different programming models of acceleration and enrichment in the area of elementary mathematics are limited and reveal mixed results. Robinson, Shore, and Enersen (2007) stated that acceleration enables students to cover content efficiently. However, they cautioned that acceleration alone does not attend to the development of the high-level mathematical thinking characteristic of talented students. Stanley, Lupkowski, and Assouline (1990) viewed acceleration as a good fit for only a small percentage of students. On the other hand, it is not an uncommon practice for programs that focus on enrichment to have students work on a "puzzle of the week" or "fun" mathematics activities, which are enjoyable but may not deepen student mathematical understanding. Sowell (1993) reviewed five studies that focused on the use of enrichment. In a study focused on elementary students, fourth graders outperformed the control groups on cognitive measures and also improved in attitudes towards mathematics. However, fifth and sixth graders were not significantly different from the control group in their achievement or attitudes towards math.

Sheffield (1999) pointed out that "services for our most promising students should look not only at changing the rate or the number of mathematical offerings but also at changing the depth or complexities of the mathematical investigations" (p. 45). Using both acceleration and enrichment as a programming model at the elementary level is promising, although only limited research has investigated this dual strategy. In one study, when exposed to a high-level curriculum that focused on developing mathematical reasoning, talented students in grades 2–7 made significant achievement gains and were satisfied with the curriculum (Robinson & Stanley, 1989). In another study, Moore and Wood (1988) found that students in grades 3–7 learned mathematics more quickly using both acceleration and enrichment than they would have if they were using the regular math curriculum. Finally, Miller and Mills (1995) found that students of varying high-ability levels in second through sixth grade made large achievement gains when placed in a program using both acceleration and enrichment. Thus, the answer to the most appropriate programming for talented elementary mathematics students may be a combination of acceleration and enrichment.

Mathematically Talented Students and Their Need for Differentiation

Mathematically talented students approach, perceive, and understand mathematics differently than other students. For instance, they are able to skip steps in the logical thought process when solving mathematical problems, can flexibly use problem-solving strategies, and have a "mathematical cast of mind" (Krutetskii, 1968/1976, p. 302). In defining mathematical promise, the Task Force on Mathematically Promising Students identified it "as a function of ability, motivation, belief, and experience or opportunity." They also stated that this definition recognized that students who are mathematically talented "have a large range of abilities and a continuum of needs that should be met" (Sheffield, 1999, p. 310).

Due to these characteristics, the curriculum must be differentiated for these students; that is, the content, process, and products used with these students consistently must be modified in response to their learning readiness and interests (Tomlinson, 1995). There are very few studies to date that study the effects of differentiation on achievement of talented elementary students. In one study with upper elementary students, Tieso (2003) found that using an enhanced or differentiated mathematics unit with above-average students from all socioeconomic backgrounds resulted in significant achievement gains compared to using a unit from the regular mathematics textbook.

Students Processing Mathematics Like Professionals

As Pelletier and Shore (2003) and Sriraman (2004) have found from their studies, mathematically talented students think about mathematics in ways similar to the ways that experts or professional mathematicians operate. Two renowned mathematicians, Jacques Hadamard (1954) and George Polya (1954), believed that the sole difference between the work of a professional mathematician and the work of a student is in the degree of sophistication they possess. Thus, both are capable of being creative and analytical in solving problems and in posing new problems at their respective levels.

In fact, encouraging students to act and work like professionals is an approach that has been in place in the field of gifted and talented education for quite a while. One of the hallmarks of the Enrichment Triad Model is the placement of students into the role of the "practicing professional" to pursue problems of particular interest to them (Renzulli, 1977). More recently Tomlinson et al. (2002) identified a special curriculum called the "Curriculum of Practice," whose intent is to provide opportunities for talented students to use the skills and methodologies of a discipline by having them function as a practicing professional in the discipline. Experts recommend using this curricular approach to help students construct and apply knowledge in a particular discipline and thus gain a deeper understanding of the subject; however, research on its effect on mathematically talented elementary students is needed.

Development of the Project M^3 Curriculum Units

In 1995, the NCTM Task Force on the Mathematically Promising urged that, "new curricula standards, programs, and materials, should be developed to encourage and challenge the development of promising mathematical students, regardless of gender, ethnicity, or socioeconomic background" (Sheffield, Bennet, Berriozabál, DeArmond, & Wertheimer, 1995, p. 8). In response, a collaborative team of experienced mathematicians, mathematics educators, and leaders in the field of gifted and talented education developed Project M^3: Mentoring Mathematical Minds curriculum units under the auspices of a U.S. Department of Education Javits Program research grant. Following the recommendations set forth in the literature, the units engage students in both advanced and enriched content as they process the mathematics like practicing mathematicians. Additionally, the lessons are differentiated to meet the range of needs of talented students. A general description of the units and how they address the literature recommendations is provided next; this is followed by a more thorough description of one unit to provide a more concrete example of how these recommendations were implemented.

Addressing the Literature Recommendations

Project M^3 has developed a total of 12 units, with 4 units at each of three levels primarily aimed at students in grades 3, 4, and 5. The content of individual units at each level is based on one of the NCTM (2000) content standards, including: (a) number and operations, (b) geometry or measurement, (c) data analysis or probability, and (d) algebra. Table 1 summarizes the primary content presented in each unit (for a more elaborate explanation of unit concepts and lessons, see Adelson & Gavin, 2006; Casa & Gavin, 2006; Casa, Spinelli, & Gavin, 2006; and Gavin, Casa, & Adelson, 2006).

The content in the units is accelerated by at least one to two grade levels. The units also are enriched with interesting and high-level mathematical investigations. One way this occurs is with an emphasis on the NCTM (2000) process standards, particularly problem solving, real-world connections, and communication. Communication provides a unique avenue for enrichment. The Project M^3 units include Chapin, O'Connor, and Anderson's (2003) talk moves (e.g., agree/disagree and why, adding on) to help teachers facilitate verbal discussions and focus on significant mathematics. Students also write about the mathematics in two in-depth and high-level written-response questions in each lesson. The verbal and written communication helps engage students' thought processes that resemble those of practicing mathematicians. "Part of learning mathematics is learning to speak

Table 1

Summary of the Content of the Project M^3 Units

NCTM Content Focus	Level 3	Level 4	Level 5
Number and Operations	In-depth exploration of our numeration system through the comparison of other systems (e.g., base 3 and additive)	Understanding of commutative and distributive properties, as well as relationships among prime, composite, square, odd, and even numbers	Development of rationale for computational algorithms and strategies to estimate, compare, and order fractions
Geometry and Measurement	Examination of length, area, volume, and surface area (e.g., applying nonstandard strategies)	Exploration of 2- and 3-dimensional shapes and their properties, the relationships among them, and spatial visualization	Investigation of similarity and changes in the perimeter, area, and volume of similar figures in relation to the scale factor
Data Analysis and Probability	Experience with collecting, analyzing, and representing data through experimental, descriptive, and historical research	Investigation of categorical and continuous data using Venn diagrams, pie and bar graphs, and line graphs	Understanding of experimental and theoretical probabilities as they relate to the Law of Large Numbers
Algebra	Study of patterns and how they change, can be extended or repeated, and grow	Solving single and sets of simultaneous equations	Exploration of rate of change, y-intercept, and intersections on tables and graphs

like a mathematician" (Pimm, 1987, p. 76). Project M^3 developed the *Student Mathematician's Journal* to support written communication. In addition to worksheets used with the investigations, the journals include two "Think Deeply" questions for each lesson to engage students in writing about significant mathematical concepts.

In accordance with prior research (Tieso, 2003; Tomlinson, 1995), Project M^3 units provide differentiated instruction so students can work at their own level of understanding. Two unique features, "Hint Cards" and "Think Beyond Cards," are available for each lesson and offer students support or further challenge when necessary. Hint Cards are for students who have had little prior experience with certain concepts and who may need a little help in getting started or moving along. Think Beyond Cards are for those students who have a firm grasp of the concepts presented in the lesson and are ready for further challenge. They ask students to expand their knowledge by using deeper, more complex reasoning.

Unit Example

Third-grade students studying *Unraveling the Mystery of the MoLi Stone: Place Value and Numeration* (Gavin, Chapin, Dailey, & Sheffield, 2006) took on the role of mathematicians at an archeological dig as they tried to decipher the numerical markings on a stone. To do this, they explored the essential concepts of place value: mainly patterns, groupings, and symbols. In the lessons, they investigated differences between place values, various bases, and other numeration systems (including the Egyptian and Chinese systems). One enrichment activity required students to apply their understandings of all of these concepts to create their own numeration systems. Acceleration occurred when students studied bases other than base-10, as these concepts typically are taught at much higher grade levels. For example, one Think Deeply question asked students to consider the similarities and differences between the base-3 and base-10 numeration systems. To help differentiate instruction, a Hint Card guided students to examine the values of the different places in a number. Similarly, at the other end of the spectrum, a Think Beyond Card had students ponder, "Does having place values in the base-10 system help us add and subtract more easily or quickly? Why or why not?"

Research Design

There were several components to the Project M^3 research study that examined the impact of the units on student achievement. Study participants included students using the Project M^3 units (intervention group) and those using the regular curriculum (comparison group). This paper addresses the efficacy of the intervention with respect to student understanding of the concepts in the units. Data collection from both the intervention and comparison groups on the Iowa Tests of Basic Skills and open-ended questions based on the released NAEP and TIMSS items is still in progress. Results on these components of the research study are forthcoming. The purpose of the study being presented was to determine if the Project M^3 units had a positive impact on the intervention group's understanding of mathematical concepts presented in the units. The following question was addressed: Was there an increase in mathematical understanding for mathematically talented students after exposure to the Project M^3 curriculum units?

Sample

The sample consisted of approximately 200 mathematically talented students entering third grade, most of whom remained in the project through fifth grade. In order to be inclusive and encourage the inclusion of typically underrepresented groups in gifted programs, Project M^3 implemented multiple methods of student identification to include minorities, second language learners, and female students as project participants. These measures included a teacher rating scale, teacher feedback on class performance and prior achievement, and a nonverbal ability test.

Student participants were from nine schools in Connecticut[1] and two schools in Kentucky. There was an almost equal breakdown by gender (50% females in grades 3 and 5 and 49% in grade 4). As indicated in Table 2, more than 40% of students

1 There were eight participating schools in Connecticut for grades 3 and 4. Students moved into a middle school in grade 5, adding a ninth school participating in the study.

Table 2

Student Demographics

Sample Size	Grade	Ethnicity/Race	Eligible for Meal Subsidy
$n = 184$	3	55% Caucasian 45% Multiethnic/racial	45%
$n = 179$	4	57% Caucasian 43% Multiethnic/racial	44%
$n = 163$	5	57% Caucasian 43% Multiethnic/racial	42%

Note. Size of the original group of Project M^3 students decreased over time due to student mobility/out-migration patterns.

were eligible for meal subsidies, and the sample was composed of students from diverse racial and ethnic groups.

Professional Development

Teachers participated in a 2-week summer training to increase their mathematical content knowledge and to implement teaching strategies developed to promote enrichment learning and mathematical communication. Teachers also attended four to six professional development sessions throughout the academic year prior to teaching each unit. These sessions included training on how to score pre- and posttests using the rubrics, as well as time to score their class sets with the supervision of the professional development team. A professional development team member visited each school every week the Project M^3 units were taught to ensure fidelity of treatment and offer individualized assistance.

Methodology

Instrumentation. Teachers in the 11 schools implemented the curriculum units (see Table 1) with Project M^3 students beginning in grade 3 and progressing through grade 5, and they administered pre- and posttests for each curriculum unit. Professional mathematics educators, in conjunction with the curriculum writers, created the tests and rubrics for each unit. They developed test questions to determine students' understanding of the major mathematical ideas in each unit. Most questions were open-response items that asked students to justify their answers (e.g., Which digit in the problem 56 + 42 should be replaced by a 7 to get the largest sum, and why?). The Project M^3 staff designed the unit rubrics to identify students' various levels of understanding (e.g., 2 points for replacing the 4 tens with the 7 and 1 point for replacing the 5 tens with the 7). They made efforts to make clear the distinction between points to be awarded by providing sample responses and identifying misconceptions.

Scoring of the Pre- and Posttests. The Project M^3 staff used the student responses on the pretests to identify approximately five samples for each question that ranged in levels of responses; they then came to a consensus about how to score them according to the rubric. They used these samples during the professional development sessions prior to the teaching of each unit to train teachers on how to score the tests.

In addition to scoring their class pretests, teachers also scored the posttests using the rubrics soon after completing each unit. Project M^3 staff double-scored all pre- and posttests. If the first and second set of scores on any subcomponent of any question did not match, another staff member triple scored it. Expert scorers discussed any other discrepancies further until a consensus was reached, thus insuring interrater agreement.

Research Results

The researchers conducted paired t tests on the total scores for each unit pre- and posttest. Table 3 summarizes student achievement gains, including the pretest and posttest mean scores with their respective standard deviations as well as effect sizes (Cohen's d calculated with pooled standard deviations) for pairs of data for all units. For each of the 12 Project M^3 units, similar results were achieved. The total scores for each unit indicate statistically significant gains from pretest mean to posttest

Table 3

Pre and Post Intervention Statistical Gains on Math Performance for the 12 Project M^3 Units

Level and Unit	n	Pretest Mean (SD)	Pretest Mean % Correct	Posttest Mean (SD)	Posttest Mean % Correct [a]	% of Students That Made Gains	t	df	d
Level 3									
Numbers and Operations	189	3.60 (3.45)	18.94	9.21 (3.78)	48.47	94	20.78**	188	1.55
Data Analysis and Probability	185	7.96 (4.02)	34.61	17.43 (3.99)	75.78	100	30.34**	184	2.36
Measurement and Geometry	180	1.31 (1.74)	6.55	11.78 (4.83)	58.90	98	30.99**	179	2.88
Algebra	176	8.54 (3.94)	35.85	16.14 (4.40)	67.25	95	22.11**	175	1.82
Level 4									
Numbers and Operations	186	2.77 (2.27)	13.85	10.47 (4.35)	52.35	98	27.05**	185	2.22
Data Analysis and Probability	160	4.83 (3.30)	21.00	16.64 (3.54)	72.35	100	41.93**	159	3.45
Measurement and Geometry	185	3.56 (2.66)	16.18	14.17 (4.49)	64.41	97	36.74**	184	2.88
Algebra	185	2.74 (2.71)	13.70	13.71 (4.06)	68.55	97	38.83**	184	3.18
Level 5									
Numbers and Operations	168	5.37 (4.07)	25.57	13.42 (4.70)	63.90	98	25.77**	167	1.83
Data Analysis and Probability	166	6.48 (2.81)	30.14	16.59 (3.53)	77.16	99	34.36**	165	3.17
Measurement and Geometry	167	2.78 (2.47)	12.64	13.96 (4.08)	63.45	99	35.72**	166	3.32
Algebra	144	5.83 (4.16)	19.43	21.52 (4.82)	71.73	99	39.27**	143	3.49

[a] Students were not expected to score 100% of the total possible points on the posttest as the researchers designed the curriculum, testing, and scoring to be very rigorous to challenge students and to avoid a ceiling effect.

$^{**}p < .01$.

mean at the $p < .01$ level of statistical significance. In addition, the effect sizes were all large (Cohen, 1965) and ranged from 1.55 to 3.49.

Students in Project M^3 began each unit with a mean pretest score ranging from 7 to 36% of the total score possible, as noted in Table 3. Although talented students typically might score higher than this on an assessment, the researchers designed the curriculum, testing, and scoring to be very rigorous to challenge students and to avoid a ceiling effect. At the end of each unit, students earned 48 to 77% of the total score, showing remarkable improvement, with mean percent total gains from 30 to 55%. The almost entirely open-ended unit tests and their rubrics required a great deal from students in explaining their answers using precise and accurate mathematics and mathematics vocabulary, and students made great strides in this process. Moreover, 94 to 100% of students, regardless of school or SES, made gains from pretest to posttest for each unit.

Discussion, Limitations, and Future Directions

The data indicate that the use of the Project M^3 units by students identified with mathematical talent produces significant gains in the understanding of the mathematical concepts outlined in the curricular units. This considerable advancement in student understanding of unit concepts occurred over a relatively short period of time (each unit took approximately 6 weeks to implement). According to Cohen (1965), an effect size equal to or greater than .80 is considered to be large, and, as noted, the effect sizes for the Project M^3 units ranged from 1.55 to 3.49. It appears that the design of the curriculum units, in combination with the professional development offerings, contributed to these findings. However, a limitation of this study is that it is not possible to isolate how much of the growth could be attributed to individual components of the units and/or the professional development support that was offered. Future investigations might explore these areas further.

As already noted, students who participated in Project M^3 represented a wide variety of racial and ethnic backgrounds. Also, almost half were eligible for meal subsidies. There was a purposeful selection of schools to ensure that the curriculum could meet the needs of those students who had already exhibited mathematical talent and those who had mathematical promise but may not have had the opportunity to demonstrate their ability. Future studies might investigate the differences between gender, racial/ethnic groups, and lower and higher socioeconomic status groups.

Other possible investigations emerge from these findings that have meaning for both researchers and teachers. What are the long-term advantages of students being exposed to the Project M^3 curriculum units? That is, will participation in Project M^3 impact their deep understanding of mathematics in middle school, high school, and beyond? Will these students select majors in mathematics and go on to become leaders in the field?

Another area of research interest is an examination of the role of verbal discussion in written communication. The researchers believe that the model of verbal discourse that was used in Project M^3 had an impact on students' understanding and ability to communicate that understanding in writing. A research study to explore this further could have far-reaching implications on the teaching and learning of mathematics.

Researchers also need to study grouping options. In most classrooms, students in this study were grouped by ability as a top group in a particular grade level and taught by one of the grade-level teachers. In two situations, a teacher of the gifted and talented, rather than a grade-level teacher, taught a class of students. The curriculum should be tested in other settings, such as cluster groups, pull-out groups, and self-contained gifted classrooms.

In conclusion, the review of literature indicates there is very little research-based curriculum that is specifically designed for or is appropriate to meet the needs of mathematically talented elementary students. This study suggests that curriculum based on the recommendations set forth in the mathematics education and gifted and talented literature can help students learn advanced mathematics. Thus, Project M^3 units may help fill this curricular void and provide an accelerated and enriched program to meet the needs of talented elementary students.

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Author Note

The work reported herein was supported under the Jacob K. Javits Gifted and Talented Students Education Act, Grant Award Number S206A020061-06 as administered by The Office of Elementary and Secondary Education. However, the findings and opinions expressed in this report do not necessarily reflect the position or policies of the U.S. Department of Education.