Mentoring Mathematical Minds: An Innovative Program to Develop Math Talent

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What are communication techniques that teachers can use to help students reach a deeper understanding of complex problems?

Earnest chatter and excited laughter is coming from the corner room at the end of the hall. Intrigued, you approach the room and notice a bold sign over the door that reads, "Our Awesome Algebra Mall, Where We Do the Coding and You Do the Solving!" Desks are lined up like a horseshoe, and student pairs sit behind them with homemade posters enthusiastically advertising their products. Your eye is drawn to the rainbow colored flowers at "Mo and Joe's Garden Shop," diary locks and keys at "Unlocking the Key," and skateboards and soccer balls at "E. T. Sports." Parents, teachers, and other students gather at each shop and try to determine the price of every item by solving the equations listed on the posters.

Mo and Joe explain to their customers that the value of each flower stays the same in every equation: 2 + = $5 and $3 = + + . The talk at "Unlocking the Key" is about how you can make an organized list to narrow down the solution for: + 2 = $11 and – $2 = instead of guessing the price of each item. You overhear a discussion about how the values or expressions on either side of the equal sign have to be the same at "E. T. Sports."

This math talk takes you back to your own experience with algebra. You recall trying to determine the values of all those *X*s and *Y*s—not always knowing how to go about it. But that was in high school, and these students are only in the 4th grade! How can this be?

Project M^3: Mentoring Mathematical Minds

The students are part of an innovative program called Project M^3: Mentoring Mathematical Minds. This program was created under a United States Department of Education Javits curriculum and research grant and is currently in its fourth year of implementation. Project M^3 is a five-year collaborative research effort of faculty at the University of Connecticut, Northern Kentucky University, and Boston University. Included in the project are teachers, administrators, and students in 11 schools of varying socioeconomic levels. A team of national experts in the fields of mathematics, mathematics education, and gifted education created a total of 12 curriculum units (four units at three different levels) of advanced mathematics that are appropriate for talented elementary students in grades 2 through 6.

The Challenge of Providing Challenge

Meeting the needs of mathematically talented elementary students has always been a real challenge due to the lack of appropriate curricular resources and training for teachers. Mathematics is not generally a strength area for elementary or gifted/talented teachers; rather, their talents and interests often lie in the language arts realm. This is coupled with the fact that, in general, elementary mathematics curriculum provides few to no advanced options for talented students. Materials labeled "enrichment worksheets" are often extra practice sheets that are neither interesting nor challenging for bright

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children. With that said, there are certainly some interesting creative problem solving books filled with logic puzzles, mind-benders, and non-routine problems, and these have a place in a challenging math program. However, they should not be **the** math program. What has been lacking and sorely needed is a curriculum that develops mathematical concepts in depth and with complexity, provides a cohesive structure in which students can explore these concepts, and requires youngsters to think and act at high levels akin to the process mathematicians use in discovering new theorems and advancements in the field.

> ***"Project-based learning provides the opportunity for choice, challenge, creativity, and real-life application, and each unit has a unique culminating project."***

A Passion for Mathematics

Project M^3 units provide such a challenge. These units engage young people to think deeply about math concepts and to make sense of them. They also engage students in interesting mathematics relevant to their lives so that they enter into a "romance with the discipline," as described by Renzulli (2002, p. 36). As a result, teachers actually see students hugging their Mathematician's Journals as they enter class eager to get started with a lesson. One student recently stated, "I used to like math, but now I love it!" In order to encourage the younger generation to accept the challenge of becoming mathematicians, a profession sorely needed in our technological society, we must instill in them a love for the subject, so that they will want to eagerly pursue this line of work.

Combining the Best of Both Worlds

So what does the mathematics in Project M^3 look like? First of all, it is not difficult arithmetic. You won't find our 3rd grade students grappling with multiplying a three-digit number by a four-digit number. A calculator can crank that out faster and with less error. Rather, we focus on the development of concepts. The content is closely aligned with the expectations that the National Council of Teachers of Mathematics (NCTM) outlined in *Principles and Standards for School Mathematics* (2000). At each of the three levels, there is a Project M^3 unit investigating important ideas from each of the content standards: number and operations, geometry and measurement, data analysis and probability, and algebra. We also have incorporated the NCTM process standards of communication, reasoning and proof, problem solving, connections, and representation.

Verbal Communication

Project M^3 encourages teachers to facilitate discussions where students reach deeper understanding of complex problems. Children explain their thinking and revise their thoughts in light of their peers' comments. They persist in their attempts to find a solution because they understand that it takes time to process and come up with different solutions. They develop the ability to organize ideas, consolidate and clarify their thinking, analyze and assess strategies, and use precise mathematical language (NCTM, 2000).

One such discussion that took place in a Project M^3 class while studying *At the Mall with Algebra: Working with Variables and Equations* (Gavin, Chapin, Sheffield, & Dailey, in press) centered on a key mathematical idea—the same variable in a set of equations has the same value in each equation. Classes investigated the following two equations that had two variables:  and . The discussions began similar to the one below.

Teacher: *Who can tell us how much they think the pencil costs? Janie?*
Janie: *I think it costs 11¢ because if the scissors costs 7¢, then 11¢ + 7¢ = 18¢.*
Teacher: *Is there someone who agrees or disagrees with that idea and can tell us why?*
Marley: *I disagree with that idea because that means that the pencil plus the pencil in the second equation would sum to 22¢, not 18¢ like it says.*
Teacher: *Derron, can you explain what Marley just said in your own words?*
Derron: *I think she said that your numbers have to work in both equations, and having the pencil cost 11¢ does not make the second equation true. Taking 11¢ plus 11¢ does not equal 18¢, so the value of the pencil cannot be 11¢.*
Teacher: *Marley, is that what you said?*
Marley: *Yes, pretty much. You have to look at both equations, not just one.*
Janie: *Oh! I just figured out that both equations would work if the pencil costs 8¢ and the scissors cost 10¢.*

As in the dialogue above, Project M^3 teachers use

talk moves (Chapin, O'Connor, & Anderson, 2003) to cognitively engage students in the process of solving mathematics problems. One talk move is *revoicing* and occurs when a teacher repeats a student's idea, then verifies with the youngster if the idea was heard accurately. For example, a teacher may state, "So you think the value of the eraser is 5¢ because 12¢ - 7¢ is 5¢? Is that what you said?" This allows teachers to clarify student contributions and encourages the class to grapple with the idea further.

A second talk move is *repeating*, which is the same as revoicing, except other children rephrase the idea. The teacher asks, "Can someone please repeat what Tyrone said in his own words?" This talk move allows students to clarify their thinking.

A third talk move is *agree/disagree and why*, which is used when students already have had time to process ideas. A teacher may ask, "Do you agree or disagree that the value of the eraser is 5¢? Please explain why." This move encourages justifying ideas with mathematical evidence.

A fourth talk move is *adding on*, which serves to expand the discussion. Teachers ask, "Who can add on to this with your own ideas?"

The final talk move is *wait time*, when teachers wait to call on someone for a response after posing a question to the class. Comments like, "We'll wait for your idea," give students time to process their responses and make them clear enough for others to understand.

Written Communication

The challenge in the Project M^3 units comes not just from advanced and comprehensive mathematics but also from written communication. Students respond to Think Deeply questions that encourage them to reflect in-depth on what they have learned about the big mathematical ideas of the units. They are encouraged to clearly express their ideas using precise mathematical language.

Before writing the equations to be used at the mall, children compared and contrasted expressions and equations. Below is a segment of what one student wrote.

An expression can be a statement that includes operations, letters or variables, and numbers, like a ÷ 2. It is not being compared to another value, so no equal sign is used. An equation is a statement that gives the value of an algebraic expression that is equal to another algebraic expression. An example of an equation is (N x 3) + 6 = 21. It uses an equal sign.

Differentiation

Unique to the Project M^3 units are Hint and Think Beyond Cards that function to support the progression of student learning. Hint Cards are used with children who need additional guidance in trying to solve a challenging problem. For example, the hint, "Now write a second equation using only one of the items and the total price," steers them towards writing a second equation that only has one variable. Think Beyond Cards are used with those needing to be challenged further. For instance, students asked to solve the equations:  = 18 and 2  = 6 are encouraged to determine the amount of pens and crayons using different strategies, such as refining their guesses after testing, using an organized list, and adding the two equations together to eliminate the variable representing the number of crayons.

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Culminating Projects

After seeing the 4th graders "at the mall" solving sets of equations, you are curious about what is happening in other Project M^3 classes and decide to visit a 5th grade classroom. As you approach, you hear laughter and applause. At the door, you are greeted by a sign for the "Carnival of Chance." The 5th grade Project M^3 students are sharing games they created with the visiting 6th graders. The laughter is coming from one particular table, at which a 6th grade student lost at a game created by a boy with a long, curly blond wig and another boy wearing a Santa hat. After a few more minutes of the Carnival of Chance, the teacher gathers both groups of students together. The boys approach the SmartBoard and use a Carroll diagram (See *en.wikipedia.org/wiki/Carroll diagram*), theoretical probability, and experimental probability to explain how their game is "slightly" unfair.

Project M^3 mathematicians sharing their work and teaching others is not unusual. Project based learning provides the opportunity for choice, challenge, creativity, and real life application, and each unit has a unique culminating project. From the cor-

rectly proportioned enlargement of Juggles the Clown to unraveling the mysterious numeration system of the MoLi Stone, students apply what they have learned throughout the units and often share their learning with parents, other students, teachers, and even superintendents. Each time they begin a unit, children enter the room eager and excited, knowing that they will use their new skills to complete a motivating culminating activity and have an opportunity to share their mathematical knowledge with others. The opportunity to work with others and apply what they have learned inspires and excites students, as illustrated by one teacher's "success story."

Ramon always wanted to sit by himself and never wanted to participate in group work or partner talk. Then one day, he was asked to design a game that was "slightly" unfair. It was explained to him that 6th graders were going to play these games and that we didn't want them to realize the games were not fair. This boy got so excited at the idea of the Carnival, and he walked around the classroom telling everyone his great ideas. He used all of his free time to work on his game, and he was beaming. He has since become the math student who joins in with other students, the student with his hand in the air first, the student always smiling with that sparkle in his eye, and the student who says, "Thank you, Mrs. M., for everything."

Evidence of Success

The teachers piloting the Project M^3 units have shared many success stories with us. These stories reveal individual student success, such as a second language student, who previously was not identified as talented, succeeding in the program and dramatically increasing her use of mathematical vocabulary in speaking and writing; a learning disabled student who was able to shine due to his mathematical insightfulness and eagerness to participate in the units; and the increased confidence of several talented girls who had previously exhibited math anxiety. They also highlight the engagement and excitement of all Project M^3 students, their sense of being a "community of mathematicians," the active participation of youngsters in mathematical discussions, and the improvement in their mathematical writing. The students' enjoyment is evident by the following comments:

- *M^3 is better than any other math class I've ever been in because some math classes [are] boring, and they don't challenge me as much. I know the answer quick[ly], but for M^3 I have to think a lot.*
- *M^3 definitely helped me in math because I used to dread going to math class, but now I'm so happy to go....Yes, it's math finally!*

Project M^3 has not only extremely positive qualitative results but also statistically significant quantitative results. Project M^3 students have made significant gains on the total scores of each unit test, the Iowa Tests of Basic Skills (ITBS), open-ended response questions from the Trends in International Mathematics and Science Study (TIMSS), and the National Assessment of Educational Progress (NAEP). These results demonstrate that mathematics knowledge and abilities improve as a result of the program. This may be due partly to the beliefs that Project M^3 teachers have in the capabilities of their students. As one father said, "I'm so truly amazed at the high expectations you have for our students. If only more adults had the same expectations for themselves." These expectations challenge children to reach new levels. As one student shared, "We got out of the comfort zone." ❖

Resources

Project M^3: Mentoring Mathematical Minds
http://www.projectm3.org

[Editor's Note: For information on all M^3 Projects, visit *www.kendallhunt.com* and search on "project m3."]

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